Assignment 1

Question 1: Let F be a field, let I be a set, and let $V = V_I$ the set of finitely supported F-valued functions on I.

a. Explain why V is an F-vector space.

b. For $i \in I$, let δ_i be the function $\delta_i : I \to F$ given by the rule

$$\delta_i(j) = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ else} \end{cases}$$

Prove that $\mathcal{B} = \{\delta_i \mid i \in I\}$ is a basis for V_I as F-vector space. You need to prove that $\mathcal{B}$ spans V and that $\mathcal{B}$ is linearly independent.

c. If $f : I \to J$ is a bijective function, prove that there is a linear mapping $\Phi_f : V_I \to V_J$ for which $\Phi_f(\delta_i) = \delta_{f(i)}$ for each $i \in I$, and that Φ_f determines an isomorphism of F-vector space $V_I \to V_J$.

Question 2: Let F be a field and let V and W be F-vector space. Suppose that $\varphi : V \to W$ is an invertible F-linear map. Using φ, define a group isomorphism $\mathrm{GL}(V) \to \mathrm{GL}(W)$.