Problem Set 1

Question 1: (*Warm up*; this question will not be graded.)

Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5\}$. Determine the following sets:

a. $A \cup B$.
b. $A \cap B$.
c. $A - B$.
d. $B - A$.
e. $A \times B$.

Question 2: For a natural number $n > 0$, consider the following intervals in the real line:

$$I_n = \left(-\frac{1}{n}, \frac{1}{n}\right) = \left\{x \in \mathbb{R} \mid -\frac{1}{n} < x < \frac{1}{n}\right\}$$

and

$$J_n = \left(n - \frac{1}{2}, n + \frac{1}{2}\right] = \left\{x \in \mathbb{R} \mid n - \frac{1}{2} < x \leq n + \frac{1}{2}\right\}.$$

a. Find $\displaystyle\bigcap_{n=1}^{\infty} I_n = \bigcap_{n \in \mathbb{N} - \{0\}} I_n$.
b. Find $\displaystyle\bigcup_{n=1}^{\infty} J_n = \bigcup_{n \in \mathbb{N} - \{0\}} J_n$.

Question 3: Prove or disprove: if U is a set with subsets $A, B \subseteq U$, then $A \cup B = A \cap B$ if and only if $A = B$.

Note: To disprove the statement, you need to provide explicitly two subsets A, B such that $A \cup B = A \cap B$ and $A \neq B$.

To prove the statement, you need to show two things: first, you must argue that

$$\text{if } A = B, \text{then } A \cup B = A \cap B;$$

second, you must argue that

$$\text{if } A \cup B = A \cap B \text{ then } A = B.$$

Question 4: Let

$$A = \{x \in \mathbb{R} \mid x^3 + 2x^2 - 3x \geq 0\} \text{ and } B = \{x \in \mathbb{R} \mid 2 - |x| < 0\}.$$

a. Describe A as a union of intervals on the real line.
b. Describe B as a union of intervals on the real line.
c. Determine $A \cap B$. Write it as a union of disjoint intervals in the real line.
d. Determine $A \cup B$. Write it as a union of disjoint intervals in the real line.